

**EXAMINATION 4**

**Directions.** Do both problems (weights are indicated). This is a closed-book closed-note exam except for four  $8\frac{1}{2} \times 11$  inch sheets containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he will not give hints and will be obliged to write your question and its answer on the board. Roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

**1.** (50 points)

Four thin rods of length  $a$  and mass  $m$  are welded together to form a square picture frame of side  $a$  and mass  $4m$ . In a set of body axes tied to the frame, the frame lies in the  $xy$  plane; its corner is at the origin and its sides lie along the  $\hat{x}$  and  $\hat{y}$  axes. (Note that the origin does not coincide with the frame's CM.)

The frame is rotating counterclockwise about the (body or space)  $\hat{y}$  axis with uniform angular velocity  $\omega$ . In the body axes, what are the components of its angular momentum?

**2.** (50 points)

Consider a football in a force-free environment. Neglecting its seams and laces, approximate it as cylindrically symmetric about its own  $\hat{3}$  (body) axis. At  $t = 0$ , a set of space ( $'$ ) axes is (momentarily) coincident with the body axes. As seen in the space axes at  $t = 0$ , the CM of the football is at rest at the origin; the football is rotating counterclockwise about the (space)  $\hat{2}'$  axis with angular velocity  $\omega$  (this is “end-over-end” rotation).

Give a qualitative description of the football's motion for  $t > 0$  as seen in its own (body) system. Justify your assertions. Does the football's angular momentum appear to stay constant when it is observed in the body system?